Section 8.5 Equations Reducible to Quadratic

Some higher exponent equations can be solved using the method of quadratics if we can make a useful substitution.

Here: $x^4 - 9x^2 + 8 = 0$ we have a 4th degree equation. Notice that if we made a substitution of $u = x^2$ that we could rewrite the original as $u^2 - 9u + 8 = 0$.

This can be unFOILed easily to give us

$$(u-8)(u-1)=0$$

$$u = 8 \qquad u = 1$$

$$x^2 = 8 \qquad x^2 = 1$$

$$x = \pm 2\sqrt{2} \qquad x = \pm 1$$

$$x \in \left\{ \pm 2\sqrt{2}, \pm 1 \right\}$$

Consider: $x - 3\sqrt{x} = 4$. Here we could use $u = \sqrt{x}$ so $u^2 = x$ and our equation

$$u^2 - 3u = 4$$

$$u^2 - 3u - 4 = 0$$

$$(u-4)(u+1)=0$$

becomes: $u = 4$ $\qquad$ $u = -1$

$\sqrt{x} = 4$ $\qquad$ $\sqrt{x} = -1$ $\qquad$ We squared each side to get to the next line.

$x = 16$ $\qquad$ $x = 1$

Because we "squared each side" we could have introduced roots that do not work. We need to check.

$x - 3\sqrt{x} = 4$	$x - 3\sqrt{x} = 4$
$16 - 3\sqrt{16} = 4$	$1 - 3\sqrt{1} = 4$
$16 - 12 = 4$	$1 - 3 = 4$
$4 = 4$	$-2 = 4$
true	false

So only good solution is $x = 16$.

Example 2, page 530

Given $f(x) = (x^2 - 1)^2 - (x^2 - 1) - 2$. Find the x-intercepts.
x-intercepts are when y (or in our case $f(x)$)= 0.

$$(x^2 - 1)^2 - (x^2 - 1) - 2 \overset{\text{set}}{=} 0$$

We let $u = x^2 - 1$ and get $u^2 - u - 2 = 0$

Factoring we get: $(u - 2)(u + 1) = 0$

$u = 2 \quad u = -1$

	$u = 2$	$u = -1$
	$x^2 - 1 = 2$	$x^2 - 1 = -1$
Then	$x^2 = 3$	$x^2 = 0$
	$x = \pm\sqrt{3}$	$x = 0$

$$x \in \{0, \pm\sqrt{3}\}$$

Example $m^{-2} - 6m^{-1} + 4 = 0$

The significant thing is that you recognize that $\left(m^{-1}\right)^2 = m^{-2}$

So you would let $u = m^{-1}$ and substitute.
Toward the end of this problem you will get $u = 3 \pm \sqrt{5}$.
That means

$m^{-1} = 3 \pm \sqrt{5}$

$\frac{1}{m} = \left(3 \pm \sqrt{5}\right)$ we have a single item on each side, we can turn the equation over.

$m = \frac{1}{3 \pm \sqrt{5}}$ now we rationalize....

$$m = \frac{1}{3 \pm \sqrt{5}} \cdot \frac{3 \mp \sqrt{5}}{3 \mp \sqrt{5}}$$

$$m = \frac{3 \mp \sqrt{5}}{9 - 5}$$

$$m = \frac{3 \mp \sqrt{5}}{4}$$

Example 5 page 532 is similar.

$$t^{\frac{2}{5}} - t^{\frac{1}{5}} - 2 = 0$$

Let

$$u = t^{\frac{1}{5}}$$

and it follows the previous example.

Section 8.6 Quadratic Functions and Their Graphs

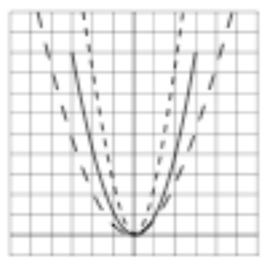
$f(x) = a(x-h)^2 + k$ is the graphing form of a quadratic function.

This is to quadratic functions what $y = mx + b$ is to straight lines.

If $a > 0$, the function opens up and thus has a minimum value.
If $a < 0$, the function opens down and has a maximum value.

The maximum (or minimum) value is called the "vertex" of the function.
The vertex has coordinates (h, k)

Comparing to the standard $f(x) = x^2$, a function with $0 < a < 1$ opens wider.
Comparing to the standard $f(x) = x^2$, a function with $a > 1$, opens narrower.

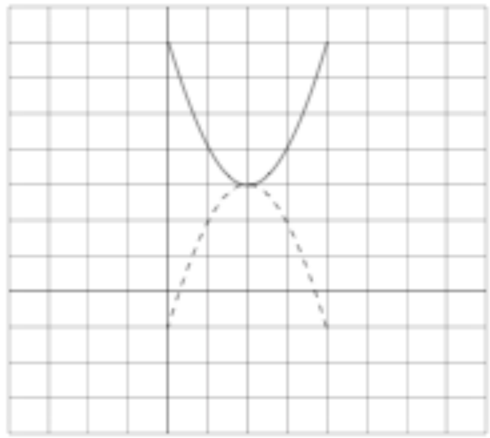


The widest graph has equation $g(x) = \frac{1}{2}x^2$.

The "middle" graph has equation $f(x) = x^2$

The narrow graph has equation $h(x) = 2x^2$

The "axis of the function" is the function's vertical center. This axis has the equation $x = h$. Both of these functions have $x = 2$ as their axis:



The top function is: $f(x) = (x-2)^2 + 3$.
The bottom function is: $g(x) = -(x-2)^2 + 3$.
Notice that the bottom function open downward since the "*a*" number is negative one.